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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION (UPDATED)

CLASS - 12 (PCM)
Question Paper Code : UN494

KEY

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SOLUTIONS

MATHEMATICS

01. (B)

02. (A) Here $A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$

$$\Rightarrow A^T = \begin{bmatrix} 2 & 4 & -5 \\ 4 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}$$

Now,

$$\Rightarrow A + A^T = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow A + A^T = \begin{bmatrix} 2+2 & 0+4 & -3-5 \\ 4+0 & 3+3 & 1+7 \\ -5-3 & 7+1 & 2+2 \end{bmatrix}$$

$$\Rightarrow A + A^T = \begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix}$$

$$A-A^T=\begin{bmatrix}2&0&-3\\4&3&1\\-5&7&2\end{bmatrix}-\begin{bmatrix}2&4&-5\\0&3&7\\-3&1&2\end{bmatrix}$$

$$\Rightarrow A-A^T=\begin{bmatrix}2-2&0-4&-3+5\\4-0&3-3&1-7\\-5+3&7-1&2-2\end{bmatrix}$$

$$\Rightarrow A-A^T=\begin{bmatrix}0&-4&2\\4&0&-6\\-2&6&0\end{bmatrix}$$

$$\text{Let } P=\frac{1}{2}\left(A+A^T\right)=\frac{1}{2}\begin{bmatrix}4&4&-8\\4&6&8\\-8&8&4\end{bmatrix}=\begin{bmatrix}2&3&-4\\2&3&4\\-4&4&2\end{bmatrix}$$

$$Q=\frac{1}{2}\left(A-A^T\right)=\frac{1}{2}\begin{bmatrix}0&-4&2\\4&0&-6\\-2&6&0\end{bmatrix}=\begin{bmatrix}0&-2&1\\2&0&-3\\-1&3&0\end{bmatrix}$$

Now,

$$P^T=\begin{bmatrix}2&2&-4\\2&3&4\\-4&4&2\end{bmatrix}^T=\begin{bmatrix}2&2&-4\\2&2&4\\-4&4&2\end{bmatrix}=P$$

$$Q^T=\begin{bmatrix}0&-2&1\\2&0&-3\\-1&3&0\end{bmatrix}=\begin{bmatrix}0&2&-1\\-2&0&3\\1&-3&0\end{bmatrix}=-\begin{bmatrix}0&-2&1\\2&0&-3\\-1&3&0\end{bmatrix}=-Q$$

$$P+Q=\begin{bmatrix}2&2&-4\\2&2&4\\-4&4&2\end{bmatrix}+\begin{bmatrix}0&-2&1\\2&0&-3\\-1&3&0\end{bmatrix}$$

$$=\begin{bmatrix}2+0&2-2&-4+1\\2+2&3+0&4-3\\-4-1&4+3&2+0\end{bmatrix}$$

$$=\begin{bmatrix}2&0&-3\\4&3&1\\-5&7&2\end{bmatrix}=A$$

Thus, we have expressed A is the sum of a symmetric and a skew-symmetric matrix.

Hence,the symmetric matrix is

$$\begin{bmatrix}2&2&-4\\2&3&4\\-4&4&2\end{bmatrix}$$

03. (B) $\tan^{-1}\frac{a}{b}-\tan^{-1}\frac{a-b}{a+b}$

$$=\tan^{-1}\left(\frac{\frac{a}{b}-\frac{a-b}{a+b}}{1+\frac{a}{b}\cdot\frac{a-b}{a+b}}\right)=\tan^{-1}\left(\frac{a^2+ab-ab+b^2}{ab+b^2+a^2-ab}\right)$$

$$=\tan^{-1}1=\frac{\pi}{4}$$

04. (C) Let, $I=\int_{\pi/6}^{\pi/3}\frac{1}{1+\sqrt{\cot x}}dx$.....(i)

$$\int_{\pi/6}^{\pi/3}\frac{1}{1+\sqrt{\cot\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}dx\left[\text{using }\int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$$

$$\int_{\pi/6}^{\pi/3}\frac{1}{1+\sqrt{\tan x}}dx \text{ .... (ii)}$$

Adding (i) and (ii) we get

$$2I=\int_{\pi/6}^{\pi/3}\left[\frac{1}{1+\sqrt{\cot x}}+\frac{1}{1+\sqrt{\tan x}}\right]dx$$

$$=\int_{\pi/6}^{\pi/3}\frac{2+\sqrt{\cot x}+\sqrt{\tan x}}{\left(1+\sqrt{\cot x}\right)+\left(1+\sqrt{\tan x}\right)}dx$$

$$=\int_{\pi/6}^{\pi/3}\left[\frac{2+\sqrt{\cot x}+\sqrt{\tan x}}{2+\sqrt{\cot x}+\sqrt{\tan x}}\right]dx$$

$$=\int_{\pi/6}^{\pi/3}dx$$

$$=[x]_{\pi/6}^{\pi/3}=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}$$

Hence, $I = \frac{\pi}{12}$

05. (D) Now, $f'(1^-) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h}$

$$= \lim_{h\to 0} \frac{(1-h-1).\sin\left(\frac{1}{1-h-1}\right) - 0}{-h}$$

$$= \lim_{h\to 0} \sin\left(-\frac{1}{h}\right) = -\lim_{h\to 0} \sin\frac{1}{h}$$

and $f'(1^+) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$$= \lim_{h\to 0} \frac{(1+h-1)\sin\left(\frac{1}{1+h-1}\right)-0}{h} = \lim_{h\to 0} \sin\frac{1}{h}$$

$\therefore$ $f'(1^-) \neq f'(1^+)$

Hence, f is not differentiable at x = 1

Again,

$$f'(0) = \lim_{h\to 0} \frac{(0+h-1)\sin\left(\frac{1}{0+h-1}\right) - \sin 1}{-h}$$

$$= \lim_{h\to 0} \frac{\left[\left\{(h-1)\cos\left(\frac{1}{h-1}\right)\times\left(\frac{-1}{(h-1)^2}\right)\right\} + \sin\left(\frac{1}{h-1}\right)\right]}{-1}$$

[using L' Hospital rule] = cos 1 − sin 1 and

$$f'(0^+) = \lim_{h\to 0} \frac{(0+h-1)\sin\left(\frac{1}{0+h-1}\right) - \sin 1}{h}$$

$$= \lim_{h\to 0} \frac{(h-1)\cos\left(\frac{1}{h-1}\right)\left(\frac{-1}{(h-1)^2}\right) + \sin\left(\frac{1}{h-1}\right)}{1}$$

[using L' Hospital rule]

= cos 1 − sin 1

$\Rightarrow$ $f'(0^-) = f'(0^+)$

Hence, f is differentiable at $x = 0$

06. (A) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$

Consider, $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = a$

$$\Rightarrow \cos^{-1}\left(\frac{x}{a}\times\frac{y}{b} - \sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{a^2}}\right)$$

$$\Rightarrow \frac{x}{a}\times\frac{y}{b} - \sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{a^2}} = \cos a$$

$$\Rightarrow \frac{x}{a}\times\frac{y}{b} - \cos a = \sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{a^2}}$$

Squaring on both sides,

$$\Rightarrow \frac{x^2y^2}{a^2b^2} + \cos^2 a - \frac{2xy}{ab}\cos a = \left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{a^2}\right)$$

$$\Rightarrow \frac{x^2y^2}{a^2b^2} + \cos^2 a - \frac{2xy}{ab}\cos a = 1 - \frac{x^2}{a^2} - \frac{y^2}{a^2} + \frac{x^2y^2}{a^2b^2}$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{2xy}{ab}\cos a = 1 - \cos^2 a$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{2xy}{ab}\cos a = \sin^2 a$$

07. (B) Put $x + \sqrt{1+x^2} = z \Rightarrow \left(1 + \frac{x}{\sqrt{1+x^2}}\right)dx = dz$

$\Rightarrow$ $z\,dx = \sqrt{1+x^2}\,dz$;

$$\therefore \int_0^\infty \frac{dx}{\left(x+\sqrt{1+x^2}\right)^n} = \int_1^\infty \frac{\sqrt{1+x^2}\,dz}{z^n.z} = \int_1^\infty \frac{\frac{1}{2}\left(z+\frac{1}{z}\right)}{z^{n+1}}dz$$

$$\left[\begin{array}{l}\therefore \left(\sqrt{1+x^2}-x\right)\left(\sqrt{1+x^2}+x\right) = 1 \\ \therefore \sqrt{1+x^2} - x = \frac{1}{z}, \therefore 2\sqrt{1+x^2} = z + \frac{1}{z}\end{array}\right]$$

$$= \frac{1}{2}\int_1^\infty \frac{z^2+1}{z^{n+2}}dz = \frac{1}{2}\int_1^\infty \left(z^{-n} + z^{-n-2}\right)dz$$

$$= \frac{1}{2}\left[\frac{z^{1-n}}{1-n} - \frac{z^{-n-1}}{(n+1)}\right]_1^\infty = \frac{n}{n^2-1}$$

08. (A) Let $f^{-1}(x) = y$ ..... (1)

$\Rightarrow f(y) = x$

$$\Rightarrow \frac{e^{y}-e^{-y}}{e^{y}+e^{-y}}=x$$

$$\Rightarrow \frac{e^{-y}\left(e^{2y}-1\right)}{e^{-y}\left(e^{2y}+1\right)}=x$$

$$\Rightarrow \left(e^{2y}-1\right)=x\left(e^{2y}+1\right)$$

$$\Rightarrow e^{2y}-1=xe^{2y}+x$$

$$\Rightarrow e^{2y}=\frac{1+x}{1-x}$$

$$\Rightarrow 2y=\log_e\left(\frac{1+x}{1-x}\right)$$

$$\Rightarrow y=\frac{1}{2}\text{loe}_e\left(\frac{1+x}{1-x}\right)$$

$$\Rightarrow f^{-1}(x)=\frac{1}{2}\text{loe}_e\left(\frac{1+x}{1-x}\right)\ [\text{from}(1)]$$

09. (D) $\tan(\cos^{-1} x) = \tan\theta$, where $\theta = \cos^{-1} x$

$$= \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-x^2}}{x}\ (x \neq 0)\ (\because \cos\theta = x)$$

10. (D) We have, $f(x) = \log_{x^2}(\log x)$

$$\Rightarrow f(x)=\frac{\log(\log x)}{\log x^2}$$

$$\Rightarrow f(x)=\frac{\log(\log x)}{2\log x}$$

$$\Rightarrow f'(x)=\frac{1}{2}\times\frac{d}{dx}\left\{\frac{\log(\log x)}{\log x}\right\}$$

$$\Rightarrow f'(x)=\frac{1}{2}\times\left\{\frac{\frac{1}{\log x}\times\frac{1}{x}\times\log x-\frac{\log(\log)x}{x}}{(\log x)^2}\right\}$$

$$\Rightarrow f'(x)=\frac{1}{2}\times\left\{\frac{\frac{1}{x}-\frac{\log(\log)x}{x}}{(\log x)^2}\right\}$$

$$\Rightarrow f'(e)=\frac{1}{2}\times\left\{\frac{\frac{1}{e}-\frac{\log(\log)e}{e}}{(\log e)^2}\right\}$$

[Putting x = e]

$$\Rightarrow f'(e)=\frac{1}{2}\times\left\{\frac{1/e}{1}\right\}$$

$$\Rightarrow f'(x)=\frac{1}{2e}$$

11. (C) Let $\Delta = \begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$

$$\Rightarrow \Delta = \begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1.1+1.1+1.1 & 1.1+1.\alpha+1.\beta & 1.1+1.\alpha^2+1.\beta^2 \\ 1.1+\alpha.1+\beta.1 & 1.1+\alpha.\alpha+\alpha.\beta & 1.1+\alpha.\alpha^2+\beta.\beta^2 \\ 1.1+1.\alpha^2+1.\beta^2 & 1.1+\alpha^2.\alpha+\beta^2.\beta & 1.1+\alpha^2.\alpha^2+\beta^2.\beta^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}^2$$

On expanding, we get

$$\Delta = (1-\alpha)^2\,(1-\beta)^2\,(\alpha-\beta)^2$$

Hence; K $(1-\alpha)^2\,(1-\beta)^2\,(\alpha-\beta)^2$

$$= (1-\alpha)^2\,(1-\beta)^2\,(\alpha-\beta)^2$$

$\therefore$ K = 1

12. (Delete)

13. (A) Let the directrix be x = – 2a and latus rectum be 4a. Then, the equation of the parabola is

(distance from focus = distance from directrix),

$x^2 + y^2 = (2a + x)^2$ or $y^2 = 4a\,(a + x)$

Differentiating w.r.t. x, we get

Putting this value of a in (1), the differential euqation is

$$y^2 = 2y\frac{dy}{dx}\left(\frac{y}{2}\frac{dy}{dx} + x\right)$$

or $$y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$$

14. (B) Points of intersection is obtained by solving y = sin$x$ and y = cos x

∴ sin$x$ = cos$x$

$$\Rightarrow x = \frac{\pi}{4}$$

Thus the two fractions intersect at

$$x = \frac{\pi}{4}$$

$$\Rightarrow y = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

Hence A $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ is the point of intersection.

∴ Area bounded by the curves and the y-axis when $0 \le x \le \pi^2$

$$A = \int_0^{\frac{1}{\sqrt{2}}} |x_1|dy + \int_{\frac{1}{\sqrt{2}}}^{1} |x_2|dy$$

$$= \int_0^{\frac{1}{\sqrt{2}}} x_1 dy + \int_{\frac{1}{\sqrt{2}}}^{1} x_2 dy$$

$$= \int_0^{\frac{1}{\sqrt{2}}} \sin^{-1} y\,dy + \int_{\frac{1}{\sqrt{2}}}^{1} \cos^{-1} y\,dy$$

$$= \left[y\sin^{-1} y + \sqrt{1-y^2}\right]_0^{\frac{1}{\sqrt{2}}} + \left[y\cos^{-1} y - \sqrt{1-y^2}\right]_{\frac{1}{\sqrt{2}}}^{1}$$

$$= \left[\frac{1}{\sqrt{2}}\sin^{-1}\frac{1}{\sqrt{2}} + \sqrt{1-\frac{1}{2}} - 1\right] +$$

$$\left[1\times\cos^{-1} - 0 - \frac{1}{\sqrt{2}}\cos^{-1}\frac{1}{\sqrt{2}} + \sqrt{1-\frac{1}{2}}\right]$$

$$= \left[\frac{1}{\sqrt{2}}\sin^{-1}\frac{1}{\sqrt{2}} + \sqrt{1-\frac{1}{2}} - 1\right]$$

$$= \left[1\times\cos^{-1} - 0 - \frac{1}{\sqrt{2}}\cos^{-1}\frac{1}{\sqrt{2}} + \sqrt{1-\frac{1}{2}}\right]$$

$$= \frac{2}{\sqrt{2}} - 1$$

$$= \left(\sqrt{2} - 1\right)\text{sq.units}$$

15. (B) Here a = 2i + j – k, b = i – j + 0. k, c = 5i – j + k.

∴ a + b – c = 2i + j – k + i – j – (5i – j + k)

= – 2i + j – 2k

∴ $|a+b-c| = \sqrt{\left\{(-2)^2 + 1^2 + (-2)^2\right\}} = 3$

∴ Unit vector in the direction of a + b – c

$$= \frac{a+b-c}{|a+b-c|} = \frac{-2i+j-2k}{3} = -\frac{1}{3}(2i-j+2k)$$

So the desired unit vector = $-\left\{-\frac{1}{3}(2i-j+2k)\right\}$

$$= \frac{1}{3}(2i - j + 2k)$$

16. (C) Given $\left(1-x^2\right)\frac{dy}{dx} - xy = 1$

$$\Rightarrow \frac{dy}{dx} - \frac{x}{1-x^2}y = \frac{1^2}{1-x}$$

Which is a linear differential equation.

$$\therefore \text{I.F.} = e^{-\int\frac{x}{1-x^2}dx}$$

Put $1 - x^2 = t$

$\Rightarrow -2x dx = dt$

$$\Rightarrow x dx = -\frac{dt}{2}$$

Now, $\text{I.F.} = e^{\frac{1}{2}\int \frac{dt}{t}}$

$$e^{\frac{1}{2}\log t} = e^{\frac{1}{2}\log(1-x^2)}$$

$$\Rightarrow \sqrt{1-x^2}$$

17. (A) We have

$$\Delta = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{vmatrix}$$

On applying $R_3 \to R_3 + R_1$

$$\Delta = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ 0 & 0 & 2 \end{vmatrix}$$

On expanding along R_3, we get

$\Delta = 2(1 + \cos^2\theta)$

$\because$ $0 \le \cos^2\theta \le 1$

$\therefore$ $1 \le 1 + \cos^2\theta \le 2$

$\Rightarrow$ $2 \le 2(1 + \cos^2\theta) \le 4$

Thus, $\Delta \in [2, 4]$

18. (C) We have

$$\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{2}$$

$$\frac{x-1}{-\sqrt{3}-1} = \frac{y-1}{\sqrt{3}-1} = \frac{z-1}{4}$$

The direction ratios of the given lines are proportional to 1, 1, 2 and $-\sqrt{3}-1, \sqrt{3}-1, 4$

The given lines are parallel to vectors

$\vec{b}_1 = \hat{i} + \hat{j} + 2\hat{k}$ and

$$\vec{b}_2 = \left(-\sqrt{3}-1\right)\hat{i} + \left(\sqrt{3}-1\right)\hat{j} + 4\hat{k}$$

Let θ be the angle between the given lines.

Now, $\cos\theta = \dfrac{\vec{b}_1 . \vec{b}_2}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}$

$$= \frac{\left(\hat{i}+\hat{j}+2\hat{k}\right).\left\{(-3-1)\hat{i}+\left(\sqrt{3}-1\right)\hat{j}+4\hat{k}\right\}}{\sqrt{1^2+1^2+1^2}\sqrt{(-3-1)^2+\left(\sqrt{3}-1\right)^2+4^2}}$$

$$= \frac{-\sqrt{3}-1+\sqrt{3}-1+8}{\sqrt{3}\sqrt{24}}$$

$$= \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

19. (B) We have 6 distinct white roses and 5 distinct red roses.

Total number of way making a garland such that no two red roses come together is

$$\frac{6! \times 5!}{2} = \frac{720 \times 120}{2} = 43200$$

20. (B) $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$

$$= \begin{vmatrix} -2y & y & y \\ x+2y & x & x+y \\ -y & 2y & -y \end{vmatrix}$$

[Applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 - R_2$]

$$y^2 = \begin{vmatrix} -2 & 1 & 1 \\ x+2y & x & x+y \\ -1 & 2 & -1 \end{vmatrix}$$

[Taking (y) common from R_1 and from R_3]

$$y^2 = \begin{vmatrix} -2 & 1 & 3 \\ x+2y & 3x+4y & -y \\ -1 & 0 & 0 \end{vmatrix}$$

[Applying $C_2 \to C_2 + 2C_1$ and $C_3 \to C_3 - C_1$]

$= y^2[-1(3y - 9x - 12y)]$

$= y^2[9y + 9x]$

$= 9y^2(y + x)$

Hence, the correct option is (B).

21. (B) $|A| = \dfrac{1}{(a+ib)(a-ib)-(c+id)(-c+id)}$

$= \dfrac{1}{a^2+b^2+c^2+d^2} = 1$

$\therefore A^{-1} = \begin{bmatrix} a-ib & -c-id \\ c-id & a+ib \end{bmatrix}$

22. (B) n = 8, p = $\dfrac{1}{2}$ = q

$P(|X-4|) \le 2$

$\Rightarrow -2 \le X - 4 \le 2$

$\Rightarrow 4 - 2 \le X \le 2 + 4$

$\Rightarrow 2 \le X \le 6$

$P(2 \le X \le 6) = P(2)+P(3)+P(4)+P(5)+P(6)$

$$P(2 \le X \le 6) = {}^8C_2\left(\frac{1}{2^8}\right) + {}^8C_3\left(\frac{1}{2^8}\right) + {}^8C_4\left(\frac{1}{2^8}\right) + {}^8C_5\left(\frac{1}{2^8}\right) + {}^8C_6\left(\frac{1}{2^8}\right)$$

$= \dfrac{119}{128}$

23. (B) $-x_1 : x_2 = 3 : 2$

24. (D) We need to maximize the function

Z = x + y

Converting the given inequations into equations, we obtain x + 2y = 70, 2x + y = 95, x = 0 and y = 0

Region represented by $x + 2y \le 70$.

The line x + 2y = 70 meets the coordinate axes at A(70, 0) and B(0, 35) respectively.

By joining these points we obtain the line x + 2y = 70.

Clearly (0, 0) satisfies the inequation $x + 2y \le 70$.

So, the region containing the origin represents the solution set of the inequation $x + 2y \le 70$.

Region represented by $2x + y \le 95$

The line 2x + y = 95 meets the coordinate axes at $C\left(\dfrac{95}{2}, 0\right)$ & D(0, 95) respectively.

By joining these points we obtain the line 2x + y = 95.

Clearly (0, 0) satisfies the inequation $2x + y \le 95$.

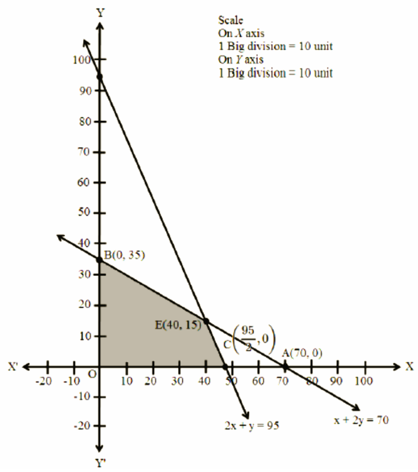
So, the region containing the origin represents the solution set of the inequation $2x + y \le 95$.

Region represented by $x \ge 0$ and $y \ge 0$.

Since, every point in the first quadrant satisfies these inequations.

So, the first quadrant is the region represented by the inequations $x \ge 0$, and $y \ge 0$.

The feasible region determined by the system of constraints $x + 2y \le 70$, $2x + y \le 95$, $x \ge 0$, and $y \ge 0$, are as follows.



The corner points of the feasible region are O(0, 0), $C\left(\dfrac{95}{2}, 0\right)$, E(40, 15) and B(0, 35).

The values of Z at these corner points are as follows

Corner point	$Z = x + y$
$O(0,0)$	$0+0=0$
$C\left(\frac{95}{2},0\right)$	$\frac{95}{2}+0,2=\frac{95}{2}$
$E(40,1)$	$40+15=55$
$B(0,35)$	$0+35=35$

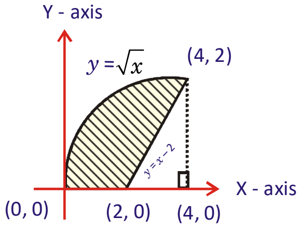
We see that the maximum value of the objective function Z is 55 which is at (40, 15).

25. (B) The intersection point of

$y = x - 2$ and $y = \sqrt{x}$ is (4, 2)

Required area

$$= \int_0^4 \sqrt{x}dx - \frac{1}{2}\times 2\times 2 = \frac{16}{3} - 2 = \frac{10}{3}$$



PHYSICS

26. (D) In series, the effective e.m.f. of cells = 3 E, effective internal resistance = 3 r

In parallel, the effective e.m.f. of cells = E, effective internal resistance = r / 3

As per question, $I = \frac{3E}{2+3r} = \frac{E}{2+r/3}$

or 6 + r = 2 + 3 r or r = 2 Ω

$\therefore \quad I = \frac{3\times 2}{2+3\times 2} = \frac{6}{8} = 0.75$ A

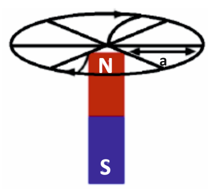
27. (D) Initially at t = 0, the inductor will act as a open circuit and no current will pass through the circuit, hence current in circuit = 0.

Magnetic field energy = $\frac{1}{2}Li^2 = 0$, since i = 0 ;

Power delivered by battery = Vi = O as i = O

And emf induced = $L\frac{di}{dt}$, since current is changing with time in inductor hence the emf induced will be non-zero.

28. (A) In this case, the north pole of a very long, bar magnet is coinciding with the centre of the circular loop carrying electric current i. So, the magnetic field lines almost lie on the plane of the ring and the force due to the field lines is perpendicular to the field lines and to the plane of the circular ring.



Let idl be the current element, B be the magnetic field and dF be the force on the current element idl.

Now, $dF = Bidl \Rightarrow F = \int_0^{2\pi a} Bidl$

$\Rightarrow F = 2\pi aiB$

Thus, the force acting on the wire is $2\pi aiB$ and it is perpendicular to the plane of the wire.

29. (B) According to Gauss' law, the term $q_{enclosed}$ on the right side of the equation $\oint_s E.dS$ = q enclosed/Ïµ0 includes the sum of all charges enclosed by the surface called (Gaussian surface).

In left side equation, the electric field is due to all the charges present both inside as well as outside the Gaussian surface.

Hence, $\vec{E}$ on LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only.

30. (C) In a hydrogen atom, electron revolving around a fixed proton nucleus have some centripetal acceleration. Therefore, its frame of reference is non-inertial. If the frame of reference, where the electron is at rest, the given expression is not true as it forms the non-inertial frame of reference.

31. (D) $m_{Ag} = m_{Cu} \times E_{Ag} / E_{Cu}$

$= 2 \times 108 / (63.6 / 2) = 6.8$ mg

32. (C) The energy required to dissociate a carbon monoxide molecule into catbon and oxygon atoms is E = 11 eV

We know that, $E = h\upsilon = 6.62 \times 10^{-34}$ J-s

υ = frequency

$\Rightarrow 11 \text{ eV} = h\upsilon$

$$\upsilon = \frac{11 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = 2.65 \times 10^{15} \text{Hz}$$

The above frequency of radiation required for the given dissociation lies in ultraviolet region.

33. (C) Here r = 40 cm = 0.4 m

$\theta = 0°$ (an axial line)

$V = 2.4 \times 10^{-5}$ J/A-m ; M = ?

$$\text{As } V = \frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2}$$

$$2.4 \times 10^{-5} = 10^{-7}\frac{M \times 1}{(0.4)^2}$$

$M = 38.4$ A-m^2

34. (D) Two images form, one at O' and the other at O''.

The rays incident on the surface above the principal axis will be refracted in the medium of R.I. μ_3 and form the image at O''.

But the rays falling below the principal axis will be refracted in the medium having R.I. μ_1 and the image will be formed at O'. Hence, two images will be formed.

35. (D) Current flowing through 2 Ω resistance is given by $I = \frac{V}{(R+r)} = \frac{2.5}{(2\,\Omega + 0.5\,\Omega)} = 1$ A

Potential difference across the internal resistance of cell

$= (0.5\ \Omega)(1\ A) = 0.5$ V

Here, capacitor is connected in parallel with 2 Ω resistance, so it will also have 2 V potential difference across it.

P.D. across 4 μF capacitor = 2.5 V – 0.5 V = 2 V

The charge on capacitor plates

Q = CV = (4 μF) × 2 V = 8 μC

36. (B) Here R = 109 Ω, $X_L = \omega L = 2\pi nL$

$= 2\pi \times 50 \times 0.5 = 50\pi$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{109^2 + (50\pi)^2} = 191.12\,\Omega$$

$$I_v = \frac{E_v}{Z} = \frac{100}{191.12} = 0.52 \text{ A}$$

37. (C) According to Brewster's law, the light reflected from the top of the glass slab gets polarised as shown in the figure. The light refracted into the glass slab and the light emerging from the glass slab is only partially polarised.

Therefore, when a polaroid is held in the path of emergent light P, and rotated about an axis passing through the centre and perpendicular to the plane of polaroid, the intensity of light shall go through a minimum but not zero for two orientations of the polaroid.

38. (A) When the switch S is closed, the effective resistance of the circuit decreases and hence, current and brightness in lamp P increases. Due to it lamp P glows with more intensity. But as lamp Q is shunted, its glow decreases.

39. (D) According to Gauss' law of electrostatics, the total electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity,

i.e., $\phi = \frac{Q_{enclosed}}{\epsilon_0}$

Thus, electric flux through a surface does not depend on the shape, size or area of a surface but it depends on the amount of charge enclosed by the surface.

In given figures the charge enclosed are same that means the electric flux through all the surfaces should be the same.

40. (C) Cut off wavelength is given by

$\lambda_{min} = \frac{hc}{eV'}$

where h = Planck's constan

c = speed of light

e = charge on an electron

V = potential difference applied to the tube

When potential difference (V) applied to the tube is doubled, cutoff wavelength (λ'_{min}) is given by

$\lambda'_{min} = \frac{hc}{e(2V)} \Rightarrow \lambda'_{min} = \frac{\lambda_{min}}{2}$

Cutoff wavelength does not depend on the separation between the filament and the target.

Thus, cutoff wavelength will be halved if the the potential difference applied to the tube is doubled.

CHEMISTRY

41. (B) $k = \frac{1}{R} \times \frac{l}{a} = \frac{1}{200} \times \frac{4}{2} = \frac{1}{100} \Omega^{-1} cm^{-1}$

$\wedge_m = k \times \frac{1000}{Molarity} = \frac{1}{100} \times \frac{1000}{0.1}$

$= 100\ \Omega^{-1}\ cm^2$

42. (A) $K_2Cr_2O_7 + 4H_2SO_4 \rightarrow K_2SO_4 + Cr_2(SO_4)_3 + 4H_2O + 3O$

or

$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$

$6Fe^{2+} \rightarrow 6Fe^{3+} + 6e^-$

43. (B) A solid is said to be crystalline if the various constituent particles (atoms, ions or molecules) of which the solid is made, are arranged in a regular, definite geometrical pattern within the solid. The sharp melting point of crystalline solids due to regular arrangement of constituent particles that are observed over a long distance in the crystal lattice.

44. (A) The correct increasing order of boiling points of given compounds is

Propan-1-ol-97 °C

Butan-2-ol-100 °C

Butan-1-ol-117.7 °C

Pentan-1-ol-13.8 °C

45. (B) Rate constant, $k = 10^{-2}\ s^{-1}$

Initial reactant conc. $[A]_o$ = 10 g

Final reactant conc. $[A]_t$ = 2.5 g

Time required t = ?

For a first order reaction,

$t = \frac{2.303}{k} \log \frac{[A]_0}{[A]_t} = \frac{2.303}{10^{-2} s^{-1}} \log \frac{10\ g}{2.5\ g}$

$= 2.303 \times 10^2 \log 4$ s

$= 230.3 \times 0.6020$ s = 138.6 s

46. (A) For oxidation at anode, two possible reactions are oxidation of chlorine and of oxygen. Out of these two, oxidation of chlorine ion is preferred because oxidation of oxygen requires overvoltage.

Chlorine is obtained by electrolysis giving out hydrogen and aqueous NaOH as byproducts.

47. (B) Due to smaller size of oxygen than carbon, C = O double bond is shorter (1.23 Å) than C = C double bond (1.34 Å).

48. (C) When excess of water is added to $BiCl_3$ solution, it is hydrolysed to form a white precipitate of Bismuth Oxychloride alongwith hydrochloric acid.

$$BiCl_3 + H_2O \rightarrow \underset{\substack{\text{Bismuth oxychloride} \\ \text{(white ppt.)}}}{BiOCl} + 2HCl$$

49. (B) 1500 cc sol. = 1500 × 1.052 g = 1578 g

Solvent = 1560 g, Solute = 18 g

$$\text{Molality} = \frac{18}{60} \times \frac{1000}{1560} = 0.192$$

50. (B) According to Hardy-Schulze law, the greater the charge on anion, the greater will be its coagulating power.

Electrolytes	Anionic part	Charge on anion
Na_2S	S^{2-}	2
Na_3PO_4	PO_4^{3-}	3
Na_2SO_4	SO_4^{2-}	2
NaCl	Cl^-	1

PO_4^{3-} has highest charge. Hence, electrolyte Na_3PO_4 will have maximum coagulating value.

51. (B) No. of unpaired electrons in $[Cr(H_2O)_6]^{3+}$ = 3

Then $\mu = \sqrt{n(n+2)}$

$= \sqrt{3(3+2)} = \sqrt{15}$

= ~4BM

No. of unpaired electrons in $[Fe(H_2O)_6]^{2+}$ = 4

Then $\mu = \sqrt{4(4+2)} = \sqrt{24}$

= ~5BM

No. of unpaired electrons in $[Zn(H_2O)_6]^{2+}$ = 0

Hence, $[Fe(H_2O)_6]^{2+}$ has the highest magnetic moment value.

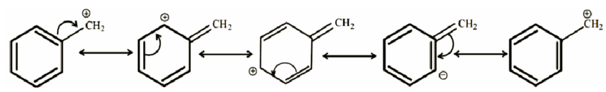
52. (C) TiO_3 behaves as conductor or insulator depending on temperature because of variation of energy gap between valence band and conduction band with the variation of temperature.

53. (D) SF_4 has distorted trigonal bipyramidal (see-saw) molecular shape with one lone pair of electrons, CF_4 has tetrahedral shape with zero lone pair of electrons and XeF_4 has square planar shape with two lone pairs of electrons.

54. (A) $C_6H_5CH_2Br$ will follow S_N1 mechanism on reaction with aqueous sodium hydroxide since the carbocation formed $C_6H_5CH_2$ is a resonance stabilized cation.

Benzylic halides show high reactivity towards the S_N^1 reaction.

The carbocation thus formed gets stabilized through resonance as shown in the structure.



55. (C) The correct order of increasing acidic strength is

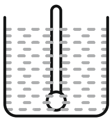
Ethanol < Phenol < Acetic acid < Chloroacetic acid.

Phenol is more acidic than ethanol because in phenol, the phenoxide ion obtained on deprotonation is stabilized by resonance which is not possible in the case of ethanol.

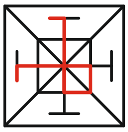
Also, carboxylic acids are more acidic than alcohols and phenols as the carboxylate ion is stabilized by resonance.

Chloroacetic acid is more acidic than acetic acid due to inductive effect of chlorine atom which stabilizes the carboxylate anion.

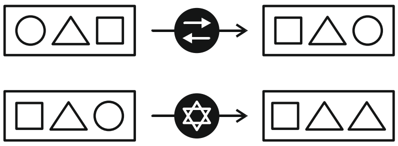
CRITICAL THINKING

56. (D) 

57. (B) Due to health precautions, businesses have promoted or shifted to contactless payment methods, leading more people to adopt and use these digital payment options.

58. (C) 

59. (A) The given statement matches the information provided, which states that Star India used the rampant illegal communication and dissemination of major sporting events on the internet as a reason for its plea to the court.

60. (B) 

The End

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